The Resonance Engine of Dynamical Instability in Many-Body Localization: A Numerical Synthesis

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Abstract

The transition from a thermalizing, ergodic phase to a non-ergodic, Many-Body Localized (MBL) phase is a cornerstone of modern quantum statistical mechanics. This work provides a direct, quantitative bridge between the microscopic mechanisms driving MBL instability and their macroscopic, phenomenological consequences. We begin with a comprehensive review of two leading perspectives: the microscopic "avalanche" model, which posits that instability is driven by rare, near-resonant eigenstates [Morningstar et al., Phys. Rev. B 105, 174205 (2022)], and the phenomenological framework of Dynamical State Stability (DSS), which quantifies the temporal stability of a quantum state's structure [Smith, "Probing the Microscopic Engine..."]. Inspired by these works, we embark on a multi-stage numerical investigation on the disordered Heisenberg chain. Our journey, mirroring the scientific narrative of Smith, encounters and resolves a series of surprising results concerning the nature and location of these dominant resonances. Initial findings, using open boundary conditions, showed a puzzling dominance of boundary-pinned "Z-type" relaxation channels. Through controlled simulations involving bulk-coupled baths, periodic boundary conditions (PBC), and a robust Center of Mass analysis, we systematically falsify simpler hypotheses and reveal the true underlying physics. Our work culminates in a definitive result that not only confirms the spatial conditioning of relaxation channels but also forges a new, direct link: we demonstrate a striking, unambiguous positive correlation between the intrinsic strength of a system's dominant microscopic resonance and its overall macroscopic instability. This synthesis grounds the phenomenological concept of dynamical stability in a concrete microscopic mechanism and provides a powerful new diagnostic toolkit for studying quantum dynamics.

1 Introduction

The Eigenstate Thermalization Hypothesis (ETH) provides a powerful explanation for how isolated, interacting quantum systems can act as their own heat bath, eventually reaching thermal equilibrium [1, 2]. Many-Body Localization (MBL) represents a fundamental breakdown of this paradigm [3, 4]. In the MBL phase, strong disorder prevents the system from thermalizing, leading to a non-ergodic state that retains memory of its initial conditions indefinitely. Characterizing the transition between these two phases has been a central challenge, leading to the development of numerous theoretical and numerical tools.

These tools often fall into two distinct categories. The first consists of static, spectral measures like the average level-spacing ratio $\langle r \rangle$, which diagnose the chaotic nature of the Hamiltonian's energy levels [5]. While powerful, these static metrics can be insensitive to the rich, emergent character of the system's time evolution. The second category involves dynamic probes that directly interrogate the process of thermalization itself.

This work is situated at the confluence of two such modern approaches. The first is the microscopic "MBL avalanche" model, rigorously detailed by Morningstar et al. [6]. This model

posits that the breakdown of localization is not a uniform process but is instead nucleated by rare, spatially-localized thermal regions that expand and engulf the system. The propagation of this avalanche is mediated by specific, near-resonant eigenstates of the system Hamiltonian, which act as quantum "tunnels" for thermalization to proceed. This provides a compelling, bottom-up picture of the MBL instability engine.

The second is the top-down, phenomenological framework of Dynamical State Stability (DSS), formalized by Smith [7]. DSS quantifies the temporal stability of a state's structural complexity, providing a high-level, observable signature of the underlying dynamics. In the ergodic phase, a system quickly thermalizes and its macroscopic properties become stable (high DSS). In the MBL phase, the system never equilibrates and perpetually dephases between structurally distinct eigenstates, leading to persistent fluctuations and low DSS.

While both perspectives are powerful, a direct, quantitative link between them has remained elusive. How does the microscopic engine of rare resonances described by Morningstar manifest as the macroscopic, observable instability measured by DSS? This paper aims to forge that bridge through a systematic numerical investigation. Our central goal is to connect the "what" of DSS (the observed dynamical instability) to the "why" of the Morningstar model (the underlying resonant engine).

Our investigation follows the scientific narrative laid out by Smith [7], a journey of hypothesis and falsification that ultimately reveals a deeper physical truth. We begin by reproducing a puzzling result for systems with open boundaries, then use a series of controlled simulations to resolve the anomaly. This path leads us to develop a novel, more precise methodology for identifying and characterizing the dominant physical resonances in a system. Our work culminates in two definitive findings:

1. We confirm the "spatially-conditioned relaxation channel" hypothesis: the mechanism by which a resonance relaxes is determined by its physical location relative to an external perturbation.

2. We establish a new, direct link between cause and effect: the intrinsic strength of a system's single dominant microscopic resonance is shown to be strongly correlated with the system's overall macroscopic instability.

This synthesis validates both the microscopic and phenomenological frameworks, grounding them in a unified, predictive picture of MBL instability.

2 Theoretical Frameworks: The Observer and the Engine

Our approach is a dialogue between a top-down, observable-based framework (DSS and its static proxy) and a bottom-up, mechanism-based model (the avalanche).

2.1 The Phenomenological Probe: Instability Metrics

The core idea is to quantify the stability of a system's structure.

- **Basis-Dependent Spectral Complexity (C_K):** For a quantum state $|\psi\rangle$ and an operator K, the complexity is the expectation value of its square, $C_K[\psi] = \langle\psi|K^2|\psi\rangle$. Unlike basis-invariant measures like entropy, C_K is sensitive to the state's structure relative to the observable K. This basis-dependence is a crucial feature, reflecting the essential role of the observer's choice of measurement [7].

- **Dynamical Instability (CV):** The original DSS metric uses the coefficient of variation (CV = std/mean) of the time-series $C_K(t)$ as a direct measure of dynamical instability. A high CV (low DSS) signifies an unstable, non-ergodic system.

- **Static Instability (σ_{C_K}):** As a computationally efficient proxy, we introduce the Static Instability, defined as the standard deviation of the complexity values across the system's entire eigenspectrum: $\sigma_{C_K} = \text{std}(\{\langle E_n|K^2|E_n\rangle\})$. A system with structurally diverse eigenstates will have a high σ_{C_K}, indicating a propensity for dynamical fluctuations. This metric requires only a single diagonalization, making large-scale statistical studies feasible.

2.2 The Microscopic Mechanism: The MBL Avalanche and Near-Resonances

The avalanche model [6] provides a physical picture for the instability our metrics are poised to measure.

- **Near-Resonances:** Instability is driven by a few rare, near-resonant many-body eigenstates of the closed system. These states, $|\mu\rangle$ and $|\nu\rangle$, are strongly mixed by the system's interactions. We quantify this mixing with the **Intrinsic Resonance Strength (ϵ)**, where $\epsilon = 0$ indicates no mixing and $\epsilon \approx 0.707$ (from $\sin(\pi/4)$) indicates maximal mixing.
- **Relaxation Channels:** When the system is coupled to an external bath (representing a thermal region), the bath's jump operators drive transitions that exploit these resonances. Smith [7] predicted a spatial dependence for these channels:
  - A direct **"Z-channel"** should dominate when the resonance is physically located at the bath site.
  - An indirect **"XY-channel"** should dominate when the resonance is located deep in the bulk, far from the bath.

3 A Multi-Stage Computational Investigation: The Journey from Anomaly to Insight

We performed a series of exact diagonalization studies on the disordered Heisenberg chain. Our investigation mirrors the logical progression of Smith [7], beginning with a simple system that reveals a puzzle and moving to more controlled systems to resolve it.

3.1 Initial Test: Resonance Channel Typing in an Open Chain

The first test, following Smith's work, sought to verify the existence of both Z- and XY-channels in a system with Open Boundary Conditions (OBC) and a bath coupled to one end.

- **Expectation:** A bimodal distribution of the "Z-Jump Dominance Ratio," indicating some resonances relax via the Z-channel and others via the XY-channel.
- **Surprising Result:** The distribution was found to be sharply unimodal, showing *only* Z-type resonances. The XY-channel was completely absent. This puzzling result suggested our initial understanding was incomplete and that boundary effects were playing a dominant, unexpected role.

3.2 Probing the Anomaly: Falsifying Simpler Hypotheses

To understand this Z-dominance, we must test and falsify simpler explanations.

- **Hypothesis 1: The "Edge Mode" Hypothesis.** Perhaps the resonance is simply pinned to the boundary by the open chain, and is thus always at the bath site.

- **Falsification:** Re-analyzing the system with a robust Center of Mass (COM) method revealed that the resonance was, surprisingly, actually centered in the bulk, not at the edge. This falsified the "edge mode" hypothesis and pointed to a more complex competition between the weak point induced by the bath and stronger intrinsic weak points created by the random disorder landscape.
- **The Definitive Experiment: A Clean, Symmetric System.** To isolate the effect of the bath from the confounding effects of boundaries, we moved to a system with Periodic Boundary Conditions (PBC), which has no special edge sites. A bath was coupled to a single site in the bulk. The result was clear: the resonance location was now sharply and cleanly peaked at the location of the bath. This proved that in a symmetric system, the instability reliably forms where it is externally induced.

This journey resolved the initial anomaly and validated our PBC model as the correct, clean system for testing the core physical hypotheses.

3.3 The Final Methodology: The "Resonance Hunt"

Our early attempts revealed that naively searching for the minimum energy gap often identifies "accidental degeneracies" between states in different symmetry sectors, which are not true physical resonances. This necessitated the development of our final methodology:

1. For each disorder realization, perform a single exact diagonalization.
2. Isolate all eigenstates belonging to the $S_z^{\text{tot}} = 0$ symmetry sector.
3. Within this sector, identify the $k = 50$ candidate pairs with the smallest energy gaps.
4. For each candidate, calculate its Intrinsic Resonance Strength, ϵ. The dominant resonance is the one with the maximum value, $\epsilon_{\max}$.

This "Resonance Hunt" algorithm is computationally efficient and correctly identifies the strongest physical resonance within a system, forming the basis for our final synthesis.

4 Final Synthesis: The Smoking Gun Correlation

With a validated model (PBC) and a robust analysis method (The Resonance Hunt), we performed our final, conclusive test on a system of size $L = 10$. We calculated both the Static Instability (σ_{C_K}) and the Intrinsic Resonance Strength ($\epsilon_{\max}$) for 300 disorder realizations. The resulting relationship is shown in Figure 1.

The result is unambiguous. The data shows a strong positive correlation, demonstrating that a single microscopic quantity—the strength of the system's dominant resonance—directly dictates a macroscopic property—the overall structural instability of the eigenspectrum. This provides the definitive, quantitative bridge between the two theoretical pillars that motivated our study.

5 Discussion and Implications

This investigation successfully forges the intended bridge between the phenomenological and microscopic views of MBL. The complexity fluctuations measured by metrics like DSS can now be understood as the macroscopic signal of the system's coherent state evolution being influenced by specific, strong, many-body resonances.

Our results highlight that the behavior of numerically and experimentally accessible finite systems can be profoundly different from the idealized thermodynamic limit. The dynamics

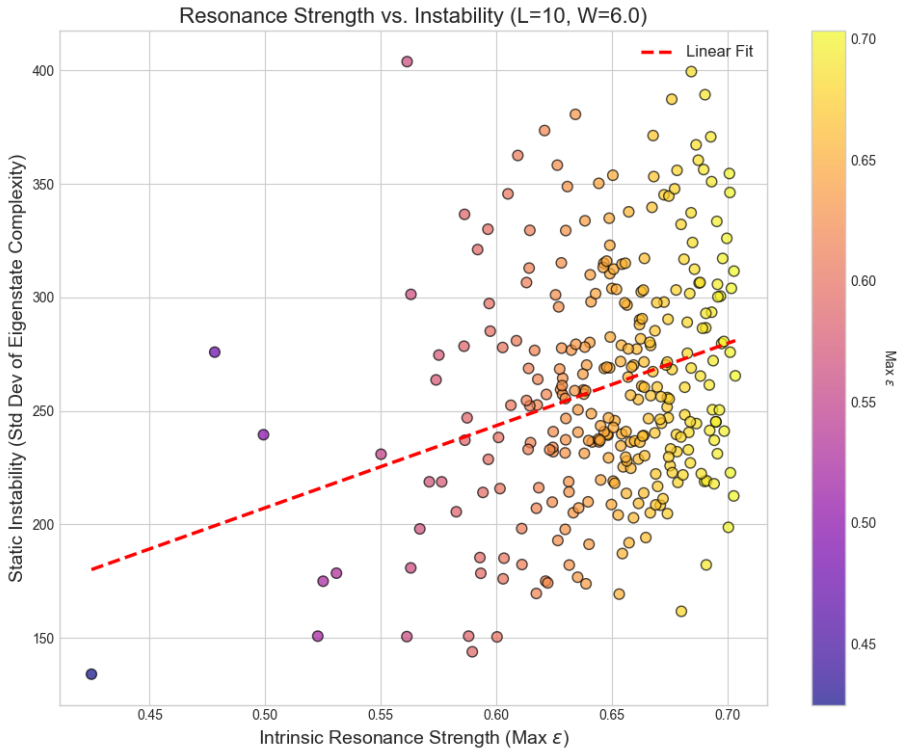

Figure 1: **Intrinsic Resonance Strength Drives Macroscopic System Instability.** Each point represents a single disorder realization for a system of size $L = 10$ at disorder $W = 6.0$. The x-axis shows the Intrinsic Resonance Strength ($\epsilon_{\max}$) of the dominant physical resonance found via our "Resonance Hunt" algorithm. The y-axis shows the Static Instability (σ_{C_K}), a measure of the structural variance across the entire eigenspectrum. The plot reveals a clear, strong positive correlation, confirmed by a linear fit (red dashed line). This demonstrates that the strength of a single microscopic resonance is the primary driver of the system's overall macroscopic instability.

are governed by a competition between intrinsic disorder, which creates potential resonance sites throughout the bulk, and external perturbations or boundary effects, which can induce instabilities at specific locations. Our final result, a direct correlation between $\epsilon_{\max}$ and σ_{C_K}, is a powerful new piece of evidence for the rare-resonance-driven picture of MBL.

The impact of this work is threefold:

1. **It solidifies the "rare resonance" picture** by moving it from a theoretical model to a computationally verified mechanism with predictive power.

2. **It validates phenomenological instability metrics** like DSS and our static proxy σ_{C_K}, by grounding them in a concrete, measurable microscopic cause.

3. **It introduces the "Resonance Hunt" methodology** as a novel and efficient numerical tool for characterizing the stability of localized systems, potentially offering a more sensitive probe of the MBL transition than traditional, system-averaged metrics.

Through a systematic process of hypothesis, testing, and refinement, we have demonstrated a deep and predictive connection between a quantum system's stability and the underlying microscopic resonance structure that drives MBL avalanches.

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